Numerical problem discussion/Classroom Activity on Properties of Fuzzy Set Operations

The Laws that Hold — with Evaluation Questions

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What we will cover

1. Properties that Hold
2. Evaluation Questions

Recall — The Three Operations

Standard fuzzy operations

$$\mu_{A\cup B} = \max(\mu_A, \mu_B), \quad \mu_{A\cap B} = \min(\mu_A, \mu_B), \quad \mu_{A'} = 1 - \mu_A$$

Key idea

Because max, min and $1-(\cdot)$ obey the same algebra as Boolean $\vee, \wedge, \neg$, *most* classical set laws carry over to fuzzy sets.

Properties (1 of 2)

Commutativity — order does not matter

$$A \cup B = B \cup A \qquad A \cap B = B \cap A$$

Example: $\max(0.8, 0.5) = 0.8 = \max(0.5, 0.8); \quad \min(0.8, 0.5) = 0.5.$

Associativity — grouping does not matter

$$(A \cup B) \cup C = A \cup (B \cup C) \qquad (A \cap B) \cap C = A \cap (B \cap C)$$

Distributivity

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

Properties (2 of 2)

Idempotence

$$A \cup A = A \qquad A \cap A = A$$

A set combined with itself gives itself back.

Involution — double complement

$$(A')' = A$$

Taking the complement twice returns the original set.

De Morgan's Laws

$$(A \cup B)' = A' \cap B' \qquad (A \cap B)' = A' \cup B'$$

Summary of Valid Properties

Property	**Statement**
Commutativity	$A \cup B = B \cup A, \quad A \cap B = B \cap A$
Associativity	$(A \cup B) \cup C = A \cup (B \cup C)$
Distributivity	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
Idempotence	$A \cup A = A, \quad A \cap A = A$
Involution	$(A')' = A$
De Morgan	$(A \cup B)' = A' \cap B'$

Now test each of these on numbers.

Question 1 — Commutativity & Associativity

Given $A = 0.8,\ B = 0.5,\ C = 0.3$

1. Verify commutativity: $A \cup B = B \cup A$ and $A \cap B = B \cap A$.
2. Verify associativity: $(A \cup B) \cup C = A \cup (B \cup C)$.

Compute each side separately, then compare.

Answer 1 — Commutativity & Associativity

(a) Commutativity

$A \cup B = \max(0.8, 0.5) = 0.8$ and $B \cup A = \max(0.5, 0.8) = 0.8.$ ✓

$A \cap B = \min(0.8, 0.5) = 0.5$ and $B \cap A = \min(0.5, 0.8) = 0.5.$ ✓

(b) Associativity

$(A \cup B) \cup C = \max(\max(0.8, 0.5), 0.3) = \max(0.8, 0.3) = 0.8.$

$A \cup (B \cup C) = \max(0.8, \max(0.5, 0.3)) = \max(0.8, 0.5) = 0.8.$ ✓

Explanation

max (and min) depend only on the *set* of values, not their order or grouping — so reordering or re-bracketing the operands never changes the result.

Question 2 — Distributivity

Given $A = 0.5,\ B = 0.8,\ C = 0.3$

Verify the distributive law

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C).$$

Evaluate the inner operation first, then the outer one.

Answer 2 — Distributivity

Left-hand side

$A \cap (B \cup C) = \min\big(0.5,\ \max(0.8, 0.3)\big) = \min(0.5, 0.8) = 0.5.$

Right-hand side

$(A \cap B) \cup (A \cap C) = \max\big(\min(0.5, 0.8),\ \min(0.5, 0.3)\big) = \max(0.5, 0.3) = 0.5.$

$$0.5 = 0.5 \quad \checkmark$$

Explanation

max distributes over min and vice versa, exactly as $\vee$ distributes over $\wedge$ in Boolean logic — so the fuzzy distributive laws hold for *any* membership values.

Question 3 — Idempotence & Involution

Given $A = 0.35$

1. Find $A \cup A$ and $A \cap A$ (idempotence).
2. Find A', then $(A')'$ (involution).

Recall $A' = 1 - \mu_A$.

Answer 3 — Idempotence & Involution

(a) Idempotence

$A \cup A = \max(0.35, 0.35) = 0.35$ and $A \cap A = \min(0.35, 0.35) = 0.35$. ✓

(b) Involution

$A' = 1 - 0.35 = 0.65$, then $(A')' = 1 - 0.65 = 0.35 = A$. ✓

Explanation

Idempotence: $\max(a, a) = \min(a, a) = a$, so a set combined with itself is unchanged. Involution: $1 - (1 - a) = a$, so complementing twice cancels out and returns the original.

Question 4 — De Morgan's Laws

Given $A = 0.6,\ B = 0.9$

Verify *both* laws:

$$(A \cup B)' = A' \cap B' \qquad (A \cap B)' = A' \cup B'.$$

Build each side: first $A \cup B$ and its complement, then A', B'.

Answer 4 — De Morgan's Laws

Law 1: $(A \cup B)' = A' \cap B'$

$A \cup B = \max(0.6, 0.9) = 0.9$

$(A \cup B)' = 1 - 0.9 = 0.1$

$A' = 0.4, \;\; B' = 0.1$

$A' \cap B' = \min(0.4, 0.1) = 0.1$

$0.1 = 0.1$ ✓

Law 2: $(A \cap B)' = A' \cup B'$

$A \cap B = \min(0.6, 0.9) = 0.6$

$(A \cap B)' = 1 - 0.6 = 0.4$

$A' = 0.4, \;\; B' = 0.1$

$A' \cup B' = \max(0.4, 0.1) = 0.4$

$0.4 = 0.4$ ✓

Explanation

Complementing flips large $\leftrightarrow$ small values, so $1 - \max = \min$ of the complements and $1 - \min = \max$ of the complements — turning union into intersection and vice versa.

Wrap-Up

What the questions confirmed

- Commutativity, associativity, distributivity, idempotence, involution and De Morgan's laws **all hold** for fuzzy sets.
- Each follows directly from the algebra of max, min and $1 - (\cdot)$.

Thank You — Questions?