Fuzzy Logic

Unit 2: Fuzzy Sets, Operations, Relations, Inference & Fuzzy Control

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Soft Computing (CSA301) — Unit 2

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Course Page:
https://www.gcjana.in/courses/shardauniversity/2601/soft-computing/

Outline

Learning Outcomes of Unit 2

After this unit, a student will be able to...

1. **Define** fuzzy sets and membership functions and contrast them with crisp sets.
2. **Apply** the standard operations (union, intersection, complement) and verify their properties.
3. **Construct** fuzzy relations and compose them (max–min composition).
4. **Formulate** fuzzy propositions, IF–THEN rules and implications.
5. **Perform** approximate reasoning using the compositional rule of inference.
6. **Defuzzify** a fuzzy output (centroid, MoM, etc.) and **design** a simple fuzzy logic controller.
7. **Relate** fuzzy logic to real-life societal applications.

Mapping to the Syllabus

A. Fuzzy logic, fuzzy sets, membership functions. **B.** Operations, relations, rules, propositions, implications, inferences. **C.** Defuzzification, controller design, societal applications.

Recap: Crisp Sets and the Characteristic Function

Definition

A **crisp** set $S \subseteq X$ has *binary* membership described by its **characteristic function**:

$$\mu_S(x) = \begin{cases} 1 & x \in S \\ 0 & x \notin S \end{cases}$$

- Every element either **belongs** or **does not**.
- Boundaries are **sharp** — no "in-between".
- This is the world of *hard computing* (Unit 1).

Example — "Tall people" (crisp)

Let *tall* $= \{h \mid h \geq 180 \text{ cm}\}$.
Then 179.9 cm is **not** tall, 180.0 cm **is** tall. Absurd at the boundary!

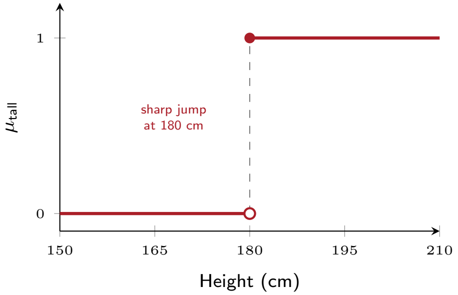


Figure: A crisp set has a discontinuous, all-or-nothing boundary.

Motivation: The World is Fuzzy

Crisp logic (two-valued)

Question: *Is the liquid colourless?*

Liquid	Answer
Water	Yes
Milk	No
Cola	No
Sprite	Yes

Only **True / False**.

Fuzzy logic (multi-valued)

Question: *Is the person honest?*

Person	Degree
Ankit	0.99 (extremely)
Rajesh	0.75 (very honest)
Santosh	0.55 (honest at times)
Kabita	0.35 (dishonest)

A **continuum** of truth $\in [0, 1]$.

Introduced by Lotfi A. Zadeh (1965) — "The closer you look at reality, the fuzzier it gets."

What is a Fuzzy Set?

Formal Definition

A **fuzzy set** A in a universe of discourse X is a set of ordered pairs

$$A = \{(x,\ \mu_A(x)) \mid x \in X\}, \qquad \mu_A : X \to [0,1],$$

where $\mu_A(x)$ is the **grade (degree) of membership** of x in A.

- $\mu_A(x) = 1 \Rightarrow x$ is **fully** in A.
- $\mu_A(x) = 0 \Rightarrow x$ is **not** in A.
- $0 < \mu_A(x) < 1 \Rightarrow$ **partial** membership.

Notation

Discrete: $A = \sum_i \mu_A(x_i)/x_i$ ("/" is a separator, not division).
Continuous: $A = \int_X \mu_A(x)/x$.

Key note

A crisp set is a *special* fuzzy set whose memberships are only 0 or 1.

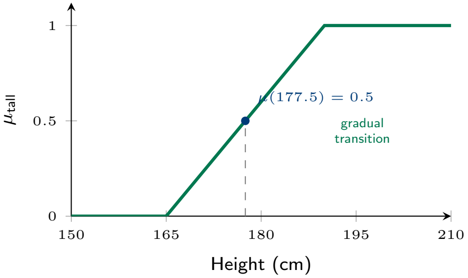


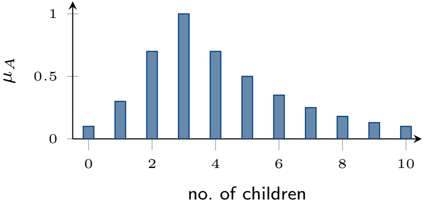
Figure: *Tall* as a fuzzy set — membership rises smoothly.

Fuzzy Set — Discrete and Continuous Universes

Discrete universe

$X = \{0, 1, \ldots, 10\}$ (number of children),
$A =$ "Happy Family".

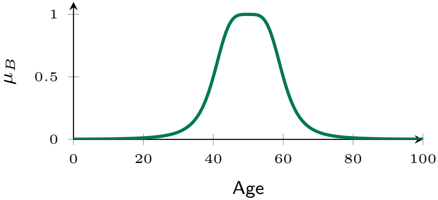
$$A = \frac{0.1}{0} + \frac{0.3}{1} + \frac{0.7}{2} + \frac{1.0}{3} + \frac{0.7}{4} + \cdots + \frac{0.1}{10}$$



Continuous universe

$X = \mathbb{R}^+$ (age), $B =$ "Middle-aged".

$$\mu_B(x) = \frac{1}{1 + \left(\frac{x - 50}{10}\right)^4}$$



The membership function *is* the fuzzy set — choosing it well is the art of fuzzy modelling.

Key Terminologies of a Fuzzy Set

Definitions

- **Support**: $\text{supp}(A) = \{x \mid \mu_A(x) > 0\}$
- **Core**: $\text{core}(A) = \{x \mid \mu_A(x) = 1\}$
- **Height**: $h(A) = \max_x \mu_A(x)$
- **Normal**: A is normal if $h(A) = 1$; else *subnormal*
- **Crossover point**: $\{x \mid \mu_A(x) = 0.5\}$
- **α-cut**: $A_\alpha = \{x \mid \mu_A(x) \geq \alpha\}$ (strong: $> \alpha$)
- **Bandwidth**: distance between the two crossover points of a normal, convex set

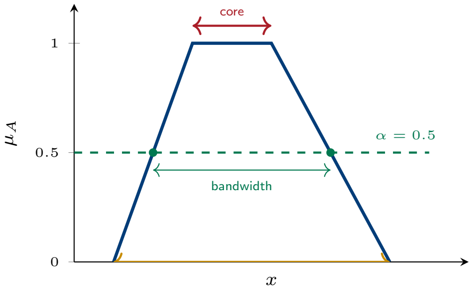
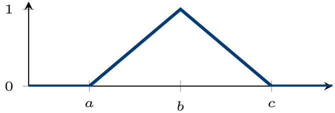


Figure: Support, core, α-cut, crossover points and bandwidth of a trapezoidal fuzzy set.

Standard Membership Functions (MFs)

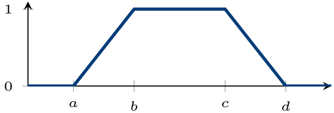
Triangular $\{a, b, c\}$

$$\mu(x) = \max\left(\min\left(\frac{x-a}{b-a},\ \frac{c-x}{c-b}\right), 0\right)$$



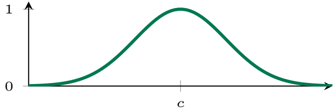
Trapezoidal $\{a, b, c, d\}$

$\mu = 1$ on $[b, c]$; linear ramps on $[a, b]$ and $[c, d]$; 0 outside.



Gaussian $\{c, \sigma\}$

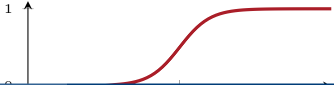
$$\mu(x) = \exp\left[-\frac{1}{2}\left(\frac{x-c}{\sigma}\right)^2\right]$$



Sigmoidal / Bell

Sigmoid: $\mu(x) = \dfrac{1}{1 + e^{-a(x-c)}}$;

Gen. Bell: $\mu(x) = \dfrac{1}{1 + \left|\frac{x-c}{a}\right|^{2b}}$.



Worked Example — Building a Triangular MF

"Warm" temperature $= \{15, 25, 35\}$

$$\mu_{\text{Warm}}(x) = \begin{cases} 0 & x \leq 15 \\ \dfrac{x-15}{10} & 15 \leq x \leq 25 \\ \dfrac{35-x}{10} & 25 \leq x \leq 35 \\ 0 & x \geq 35 \end{cases}$$

Evaluate

- $\mu_{\text{Warm}}(20) = \dfrac{20-15}{10} = \mathbf{0.5}$
- $\mu_{\text{Warm}}(28) = \dfrac{35-28}{10} = \mathbf{0.7}$
- $\mu_{\text{Warm}}(25) = \mathbf{1.0}$ (the core / peak)
- $\mu_{\text{Warm}}(40) = \mathbf{0}$

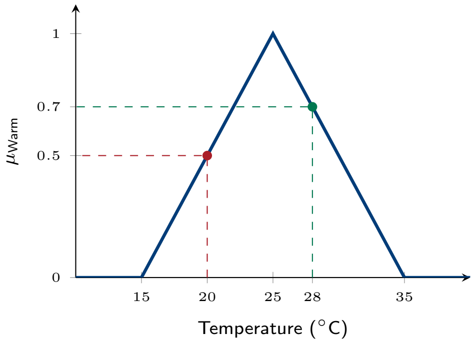


Figure: Reading membership grades straight off the triangle.

Quick Check — Round 1 (Fuzzy Sets & MFs)

[Q1]

Give the one-line difference between a crisp set and a fuzzy set.

[Q2]

For the triangular MF *Warm*$= \{15, 25, 35\}$, compute $\mu_{\text{Warm}}(30)$.

[Q3]

Define *support*, *core* and α-*cut* of a fuzzy set.

[Q4] True/False

"Every crisp set is a fuzzy set, but not every fuzzy set is crisp."

Think for a minute — answers on the next slide.

Answers — Round 1

[A1]

A crisp set uses **binary** membership $\mu \in \{0, 1\}$ (belongs or not); a fuzzy set allows a **degree** of membership $\mu \in [0, 1]$.

[A2]

30 lies on the falling edge: $\mu_{\text{Warm}}(30) = \dfrac{35 - 30}{10} = \mathbf{0.5}$.

[A3]

Support $= \{x : \mu(x) > 0\}$ (all partially-belonging elements); *core* $= \{x : \mu(x) = 1\}$ (fully-belonging elements); α*-cut* $A_\alpha = \{x : \mu(x) \geq \alpha\}$ (a crisp set for a chosen threshold α).

[A4]

True. A crisp set is the special case where all membership values are exactly 0 or 1.

Standard Operations on Fuzzy Sets

Zadeh's operators

$$\text{Complement: } \mu_{\bar{A}}(x) = 1 - \mu_A(x)$$
$$\text{Union: } \mu_{A\cup B}(x) = \max\{\mu_A(x), \mu_B(x)\}$$
$$\text{Intersect.: } \mu_{A\cap B}(x) = \min\{\mu_A(x), \mu_B(x)\}$$

Others

- Bounded sum: $\min\{1, \mu_A + \mu_B\}$
- Bounded diff.: $\max\{0, \mu_A - \mu_B\}$
- Concentration A^2 ("very"): $[\mu_A]^2$
- Dilation $A^{0.5}$ ("more or less"): $[\mu_A]^{0.5}$

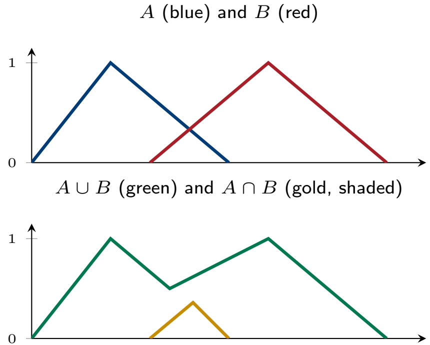


Figure: Union is the upper envelope; intersection the lower.

Worked Example — Operations on Discrete Fuzzy Sets

Given

On $X = \{1,2,3,4,5\}$: $A = \frac{0.2}{1} + \frac{0.5}{2} + \frac{0.8}{3} + \frac{1.0}{4} + \frac{0.4}{5}$, $B = \frac{0.6}{1} + \frac{0.9}{2} + \frac{0.3}{3} + \frac{0.2}{4} + \frac{0.7}{5}$.

x	1	2	3	4	5
μ_A	0.2	0.5	0.8	1.0	0.4
μ_B	0.6	0.9	0.3	0.2	0.7
$A \cup B = \max$	**0.6**	**0.9**	**0.8**	**1.0**	**0.7**
$A \cap B = \min$	**0.2**	**0.5**	**0.3**	**0.2**	**0.4**
$\bar{A} = 1 - \mu_A$	**0.8**	**0.5**	**0.2**	**0.0**	**0.6**
A^2 (very A)	0.04	0.25	0.64	1.00	0.16

Check — the excluded middle fails

At $x = 2$: $\min\{\mu_A, \mu_{\bar{A}}\} = \min\{0.5, 0.5\} = 0.5 \neq 0$. Hence $A \cap \bar{A} \neq \varnothing$ — **a fuzzy set overlaps its own complement**.

Properties of Fuzzy Set Operations

These still hold

- Commutativity, Associativity
- Distributivity
- Idempotence: $A \cup A = A$, $A \cap A = A$
- Involution: $\overline{(\bar{A})} = A$
- De Morgan's laws:
  $\overline{A \cup B} = \bar{A} \cap \bar{B}$,
  $\overline{A \cap B} = \bar{A} \cup \bar{B}$

These FAIL for fuzzy sets

- **Law of excluded middle**:
  $A \cup \bar{A} \neq X$ in general
- **Law of contradiction**:
  $A \cap \bar{A} \neq \varnothing$ in general

Why it matters

This single difference is what lets fuzzy logic represent *partial truth* — a statement can be "somewhat true and somewhat false" at once.

Verify De Morgan on the previous example at $x = 1$: $\overline{A \cup B}(1) = 1 - 0.6 = 0.4$;
$\bar{A} \cap \bar{B}(1) = \min\{0.8, 0.4\} = 0.4$. ✓

Fuzzy Relations

Definition

A **fuzzy relation** R from X to Y is a fuzzy set on the product $X \times Y$:

$$R = \{((x, y), \mu_R(x, y)) \mid (x, y) \in X \times Y\}.$$

$\mu_R(x, y) \in [0, 1]$ measures the *strength of association* between x and y.

Example — "x is close to y"

A relation on cities: $\mu_R(\text{Delhi}, \text{Noida}) = 0.9$, $\mu_R(\text{Delhi}, \text{Mumbai}) = 0.1$.

Cartesian product (rule "A and B")

$$\mu_{A \times B}(x, y) = \min\{\mu_A(x), \mu_B(y)\}$$

Relation matrix R

μ_R	y_1	y_2	y_3
x_1	0.8	0.1	0.0
x_2	0.3	1.0	0.6
x_3	0.0	0.4	0.9

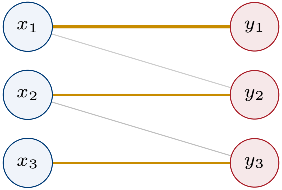


Figure: A fuzzy relation as a weighted bipartite graph.

Composition of Fuzzy Relations (Max–Min)

Definition

Given R on $X \times Y$ and S on $Y \times Z$, their **max–min composition** $R \circ S$ on $X \times Z$ is

$$\mu_{R\circ S}(x,z) = \max_{y\in Y}\ \min\{\mu_R(x,y),\ \mu_S(y,z)\}.$$

Data

$$R = \begin{pmatrix} 0.3 & 0.8 \\ 0.6 & 0.9 \end{pmatrix}, \quad S = \begin{pmatrix} 0.5 & 0.9 \\ 0.4 & 1.0 \end{pmatrix}$$

Compute entry $(1,1)$

$$\max\{\min(0.3,0.5), \min(0.8,0.4)\}$$
$$= \max\{0.3,\ 0.4\} = \mathbf{0.4}$$

Full result

$$(1,1) = \max\{\min(0.3,0.5), \min(0.8,0.4)\} = 0.4$$
$$(1,2) = \max\{\min(0.3,0.9), \min(0.8,1.0)\} = 0.8$$
$$(2,1) = \max\{\min(0.6,0.5), \min(0.9,0.4)\} = 0.5$$
$$(2,2) = \max\{\min(0.6,0.9), \min(0.9,1.0)\} = 0.9$$

$$R \circ S = \begin{pmatrix} 0.4 & 0.8 \\ 0.5 & 0.9 \end{pmatrix}$$

Composition is the engine of fuzzy inference (next).

Linguistic Variables, Propositions & Hedges

Linguistic variable

A variable whose **values are words**, not numbers.

$$T(\text{age}) = \{\text{young, old, very young, not old}, \ldots\}$$

Each term is a fuzzy set over the base variable (here, years).

Fuzzy proposition

"x **is** A" has a truth value $= \mu_A(x) \in [0, 1]$.
Connectives: AND $= \min$, OR $= \max$, NOT $= 1 - \mu$.

Linguistic hedges (modifiers)

very $A = \mu_A^2$ (concentration)
more or less $A = \mu_A^{0.5}$ (dilation)

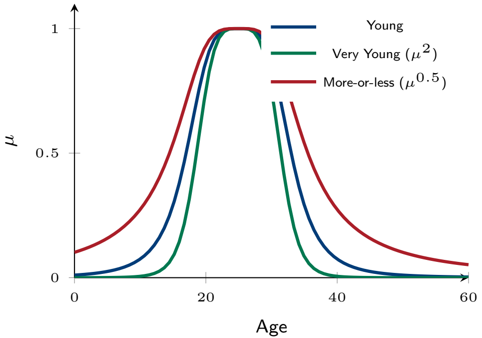


Figure: Hedges reshape the same base fuzzy set — "very" sharpens, "more or less" broadens.

Fuzzy IF–THEN Rules and Implication

Rule form

A **fuzzy rule** is a conditional statement

$$\textbf{IF } x \text{ is } A \textbf{ THEN } y \text{ is } B,$$

read as a fuzzy relation $R = A \to B$ on $X \times Y$.

Two common implications

- **Mamdani (min)**:
  $\mu_R(x, y) = \min\{\mu_A(x), \mu_B(y)\}$
- **Larsen (product)**:
  $\mu_R(x, y) = \mu_A(x) \cdot \mu_B(y)$

These *coupling* forms dominate in fuzzy *control*.

Example rule

IF temperature is **Cold** *THEN* heater is **High**.

- Antecedent fuzzy set: *Cold*
- Consequent fuzzy set: *High*
- A rule **base** is a set of such rules covering the input space.

Note

In logic, "$A \to B$" can also use S-implications (e.g.

Classical vs. fuzzy

Classical modus ponens:
Premise: IF x is A THEN y is B; x is A
Conclude: y is B.

Generalised (fuzzy):
Given a fact "x is A'" (close to A), infer

$$B' = A' \circ R, \qquad \mu_{B'}(y) = \max_{x} \min\{\mu_{A'}(x), \mu_R(x,y)\}.$$

This is the compositional rule of inference (CRI)

The observed fuzzy input is *composed* with the rule relation to yield the fuzzy output.

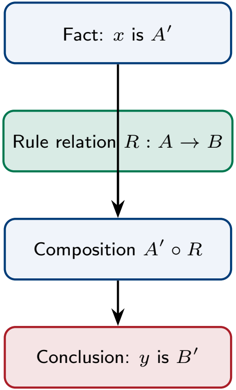


Figure: The compositional rule of inference.

If $A' = A$ exactly, fuzzy CRI reduces to the classical result $B' = B$.

Quick Check — Round 2 (Operations, Relations, Inference)

[Q1]

$\mu_A(x) = 0.7$, $\mu_B(x) = 0.4$. Find $\mu_{A\cup B}$, $\mu_{A\cap B}$ and $\mu_{\bar{A}}$ at x.

[Q2]

Which two classical laws *fail* for fuzzy sets? Give the reason.

[Q3]

Write the formula for max–min composition $\mu_{R\circ S}(x, z)$.

[Q4]

Express the hedge "*very*" as an operation on a membership function, and apply it to $\mu = 0.6$.

Answers — Round 2

[A1]

$\mu_{A\cup B} = \max\{0.7, 0.4\} = $ **0.7**; $\quad \mu_{A\cap B} = \min\{0.7, 0.4\} = $ **0.4**; $\quad \mu_{\bar{A}} = 1 - 0.7 = $ **0.3**.

[A2]

The **law of excluded middle** ($A \cup \bar{A} = X$) and the **law of contradiction** ($A \cap \bar{A} = \varnothing$). Because μ can be strictly between 0 and 1, $\min\{\mu, 1-\mu\}$ can be nonzero — a set overlaps its complement.

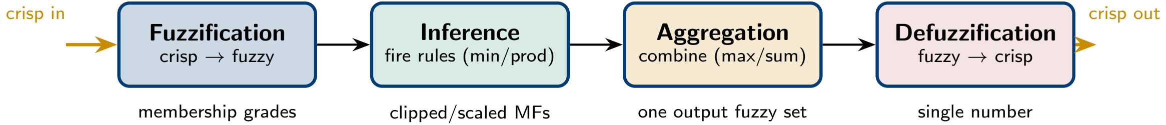
[A3]

$\mu_{R\circ S}(x,z) = \max_y \ \min\{\mu_R(x,y),\ \mu_S(y,z)\}$.

[A4]

very $A = \mu_A^2$ (concentration). At $\mu = 0.6$: $0.6^2 = $ **0.36** — membership drops, as "very" is more demanding.

The Fuzzy Inference Pipeline



Two dominant fuzzy models

Mamdani: consequents are *fuzzy sets* $\Rightarrow$ needs defuzzification (intuitive, used in control). **Sugeno (TSK)**: consequents are *crisp functions* of the inputs $\Rightarrow$ output is a weighted average (efficient; detailed in Unit 3).

Why Defuzzify? & The Main Techniques

The need

Inference produces a *fuzzy* output set, but an actuator needs **one crisp number** (e.g. "fan speed $= 42$ rpm"). **Defuzzification** maps the aggregated fuzzy set to a single value.

Common methods

- **Centroid / Centre of Gravity (CoG)** — balance point of the area
- **Bisector** — vertical line splitting area in two equal halves
- **Mean of Maximum (MoM)** — average of the peak-membership points
- **Smallest / Largest of Maximum** (SoM / LoM)
- **Weighted average** (Sugeno)

Formulas

Continuous centroid:

$$y^* = \frac{\displaystyle\int y\,\mu(y)\,dy}{\displaystyle\int \mu(y)\,dy}$$

Discrete centroid:

$$y^* = \frac{\displaystyle\sum_i y_i\,\mu(y_i)}{\displaystyle\sum_i \mu(y_i)}$$

Rule of thumb

CoG is *smooth* and most popular; MoM is *faster* but coarser. Choose by application, not by fashion.

Worked Example — Centroid Defuzzification

Discrete output fuzzy set

Fan-speed universe (rpm) with aggregated grades:

y_i	20	40	60	80	100
$\mu(y_i)$	0.2	0.6	0.9	0.4	0.1

Apply the centroid

$$\sum y_i\mu_i = 20(0.2) + 40(0.6) + 60(0.9) + 80(0.4) + 100(0.1) = 4 + 24 + 54 + 32 + 10 = 124$$

$$\sum \mu_i = 0.2 + 0.6 + 0.9 + 0.4 + 0.1 = 2.2$$

$$y^* = \frac{124}{2.2} \approx \textbf{56.4 rpm}$$

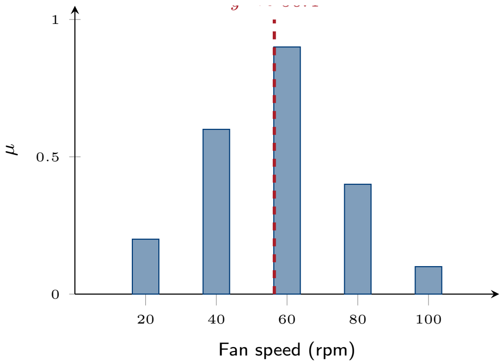
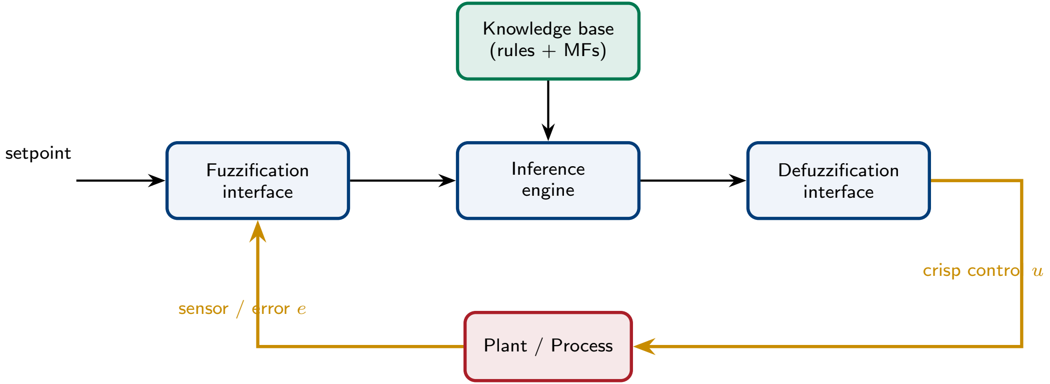


Figure: The centroid is the “balance point” of the output distribution.

Anatomy of a Fuzzy Logic Controller (FLC)



Design steps

(1) Identify input/output variables; (2) define linguistic terms & membership functions; (3) build the rule base from expert knowledge; (4) choose inference (Mamdani/Sugeno); (5) choose a defuzzifier; (6) **tune** MFs and rules by testing.

Design Walk-Through — A Fuzzy Air-Conditioner

Variables & terms

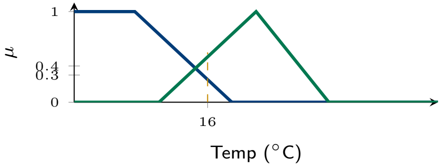
Input *Temperature*: {Cold, Cool, Pleasant, Warm, Hot}
Output *Fan speed*: {Stop, Slow, Medium, Fast, Blast}

Rule base (excerpt)

R1: IF Temp is **Cold** THEN Fan is **Stop**
R2: IF Temp is **Cool** THEN Fan is **Slow**
R3: IF Temp is **Pleasant** THEN Fan is **Medium**
R4: IF Temp is **Warm** THEN Fan is **Fast**
R5: IF Temp is **Hot** THEN Fan is **Blast**

Trace for $T = 16°\text{C}$

1. **Fuzzify**: $\mu_{\text{Cool}}(16) = 0.3$, $\mu_{\text{Pleasant}}(16) = 0.4$
2. **Infer**: R2 fires at 0.3 →Slow; R3 fires at 0.4 →Medium
3. **Aggregate**: clip Slow at 0.3, Medium at 0.4; take max
4. **Defuzzify (CoG)** ⇒ a crisp fan speed between Slow and Medium



Two-Input FLC — The Rule Matrix (FAM)

Fuzzy Associative Memory

A 2-input controller (e.g. *error* e and *change of error* Δe) is compactly written as a rule **matrix**. Each cell is the consequent term for that input combination.

Reading a cell

IF e is **NB** AND Δe is **ZE** THEN u is **NB**.
(*NB* = negative big, *ZE* = zero, *PB* = positive big.)

This is exactly how a fuzzy PID-like controller is specified — no plant model needed.

$\Delta e \rightarrow$

$e \downarrow$	NB	NS	ZE	PS	PB
NB	NB	NB	NM	NS	ZE
NS	NB	NM	NS	ZE	PS
ZE	NM	NS	ZE	PS	PM
PS	NS	ZE	PS	PM	PB
PB	ZE	PS	PM	PB	PB

Figure: A 5×5 FAM table — 25 rules on one page.
Diagonal symmetry $\Rightarrow$ balanced control action.

Fuzzy vs. Conventional (PID) Control

Conventional PID

$$u(t) = K_P e(t) + K_I \int_0^t e\, d\tau + K_D \frac{de}{dt}$$

- Needs a **precise plant model**.
- Struggles with non-linear / ill-modelled plants.

Fuzzy controller

- Encodes **expert linguistic rules**; *no explicit model*.
- Naturally handles non-linearity and multiple inputs.
- Mamdani (linguistic) or Sugeno (computationally light).

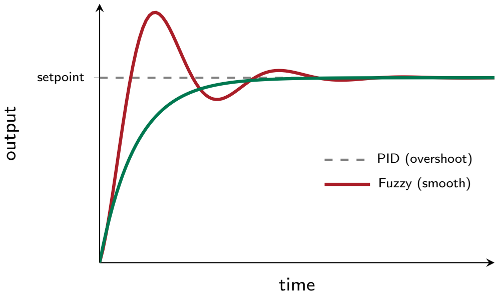


Figure: A well-tuned fuzzy controller can reach the setpoint with less overshoot on a non-linear plant.

Fuzzy Logic vs. Probability — Don't Confuse Them

Aspect	**Fuzzy Logic**	**Probability**
Models	**Vagueness / imprecision**	**Randomness / uncertainty**
Statement	"The patient has a *mild* fever"	"60% chance India wins"
Nature of category	The *category* itself is vague	The outcome is crisp; the *chance* varies
Value means	Degree of *belonging*	Frequency / likelihood of an *event*
Sums to 1?	No — memberships need not sum to 1	Yes — probabilities of a partition sum to 1
Based on	Expert judgement / perception	Recorded data / axioms

One-line memory hook

Fuzzy answers "**how much** does it belong?"; *Probability* answers "**how likely** is it to occur?" — they are complementary, not rival.

Societal Applications of Fuzzy Logic (1)

Home & consumer

- Washing machines: load/dirt sensing → water, time
- Air conditioners & refrigerators: smooth, energy-saving control
- Cameras: fuzzy auto-focus and exposure
- Rice cookers, microwave ovens

Transport & infrastructure

- **Sendai Subway (1987)** — fuzzy automatic train operation
- Fuzzy ABS, automatic transmission
- **Traffic-signal control** in congested cities
- Elevator (lift) group scheduling

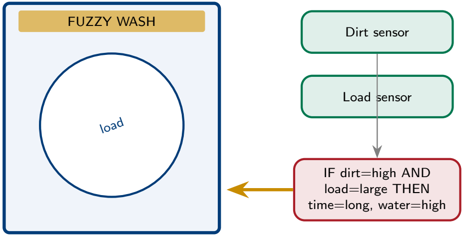


Figure: Sensor → fuzzy rules → actuator, the pattern behind most consumer fuzzy products.

Societal Applications of Fuzzy Logic (2)

Health & public services

- **Medical diagnosis** from vague symptoms (“mild”, “severe”) — remains *explainable* to clinicians
- Drug-dosage & anaesthesia-depth control
- Disaster warning: flood & fire risk indices
- Water-treatment and irrigation control

Industry & environment

- Cement kilns, steel furnaces, chemical processes
- Power-system load & voltage regulation
- Air-quality indexing, weather nowcasting
- Fuzzy decision support for policy & resource allocation

Why fuzzy suits society-scale problems

- Human judgements are **linguistic**, not numeric.
- Data is **noisy, partial and uncertain**.
- Decisions must be **transparent** and defensible.
- “Good enough, robust and cheap” beats “exact but impossible”.

Local design prompt

A fuzzy **traffic controller** for a Greater Noida junction reads *queue length* and *waiting time* to extend the green — adapting to real traffic a fixed timer cannot.

Advantages & Limitations of Fuzzy Logic

Advantages

- Captures **human/expert knowledge** directly as rules.
- **No precise model** of the plant required.
- Handles **non-linear, imprecise** systems gracefully.
- **Transparent & interpretable** (unlike a neural black box).
- Cheap sensors, low computational cost.
- Robust — **graceful degradation**.

Limitations

- Rules & membership functions need **expert tuning** (trial-and-error).
- No built-in **learning** — MFs don't self-adapt (motivates ANFIS, Unit 3).
- No formal **stability proof** in general.
- Rule count **explodes** with many inputs (curse of dimensionality).
- Choice of operators/defuzzifier is **application-specific**.

Bridge to Unit 3: couple fuzzy rules with neural learning ⇒ *ANFIS* — rules that tune themselves.

Quick Check — Round 3 (Defuzzification & Control)

[Q1]

Why is defuzzification necessary in a Mamdani controller?

[Q2]

For outputs $\mu(20) = 0.4$, $\mu(40) = 0.8$, $\mu(60) = 0.2$, compute the centroid.

[Q3]

Name the four functional blocks of a fuzzy logic controller in order.

[Q4] MCQ

Which is *not* a defuzzification method?
(a) Centroid (b) Mean of Maximum (c) Bisector (d) Backpropagation

Answers — Round 3

[A1]

Inference yields a *fuzzy* output set, but the actuator needs a single **crisp** command; defuzzification collapses the fuzzy set to one number.

[A2]

$$y^* = \frac{20(0.4) + 40(0.8) + 60(0.2)}{0.4 + 0.8 + 0.2} = \frac{8 + 32 + 12}{1.4} = \frac{52}{1.4} \approx \mathbf{37.1}.$$

[A3]

Fuzzification → **Inference engine** (with the *knowledge base*) → **Aggregation** → **Defuzzification**.

[A4]

(d) Backpropagation — that is a neural-network training rule, not a defuzzification method.

Summary of Unit 2

What we covered

1. **Fuzzy sets** — graded membership $\mu(x) \in [0, 1]$ generalising crisp sets.
2. **Membership functions** — triangular, trapezoidal, Gaussian, bell, sigmoid; support, core, α-cut, bandwidth.
3. **Operations** — union (max), intersection (min), complement; De Morgan holds, excluded middle fails.
4. **Relations** — fuzzy relations and max–min composition.
5. **Rules & inference** — linguistic variables, hedges, IF–THEN implication, generalised modus ponens (CRI).
6. **Defuzzification** — centroid, MoM, bisector; the fuzzy inference pipeline.
7. **Fuzzy control** — FLC anatomy, FAM rule matrix, fuzzy vs. PID.
8. **Applications** — appliances, transport, medicine, industry, public services.

Fuzzy logic lets machines reason with words the way humans do.

Practice Questions (Exam Oriented)

Short answer (2–5 marks)

1. Define a fuzzy set and its membership function. How does it differ from a crisp set?
2. State and illustrate the standard fuzzy operations (union, intersection, complement).
3. Which classical laws fail for fuzzy sets, and why?
4. Define support, core, height, normality and α-cut.
5. Distinguish fuzzy logic from probability with one example each.

Long answer (8–10 marks)

6. Given two discrete fuzzy sets, compute union, intersection, complement and “very”.
7. Explain max–min composition and solve a 2×2 numerical example.
8. Describe the fuzzy inference pipeline and defuzzify a given output by the centroid.
9. Draw and explain the block diagram of a fuzzy logic controller; design a rule base for a fuzzy air-conditioner.
10. Discuss four societal applications of fuzzy logic, stating why fuzzy suits each.

Think & Explore (Take-Home)

Mini design task

Design (on paper) a **fuzzy fan/heater controller**:

- Input: *room temperature* (5 terms)
- Output: *heater power* (5 terms)
- Sketch the membership functions
- Write 5 rules and defuzzify one case by the centroid

Coding (self-learning)

Implement the same controller in Python using `scikit-fuzzy` (`skfuzzy`) and compare with your hand calculation.

Reading & reflection

- Klir & Yuan, chapters on fuzzy sets, operations and relations.
- Ross, *Fuzzy Logic with Engineering Applications* — controller design.
- Prepare for **Unit 3**: Mamdani, Sugeno, Tsukamoto models and ANFIS.

Reminder

Assignment 1 covers Units 1 & 2 — handwritten, step-by-step, submitted as a single PDF via the course page.

References

Textbooks (as per the course page)

1. G. J. Klir and B. Yuan, *Fuzzy Sets and Fuzzy Logic: Theory and Applications*, Prentice Hall. **[Main text]**
2. T. J. Ross, *Fuzzy Logic with Engineering Applications*, McGraw Hill.
3. J.-S. R. Jang, C.-T. Sun and E. Mizutani, *Neuro-Fuzzy and Soft Computing*, Prentice Hall.

Seminal papers

4. L. A. Zadeh, "Fuzzy Sets," *Information and Control*, 8(3):338–353, 1965.
5. L. A. Zadeh, "Outline of a New Approach to the Analysis of Complex Systems and Decision Processes," *IEEE Trans. SMC*, 1973.
6. E. H. Mamdani and S. Assilian, "An experiment in linguistic synthesis with a fuzzy logic controller," *Int. J. Man-Machine Studies*, 1975.

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Thank You

Any Questions?

"As complexity rises, precise statements lose meaning and meaningful statements lose precision."

— Lotfi A. Zadeh (the Principle of Incompatibility)

Post your doubts on the course page → gcjana.in/. . . /soft-computing/#Post-doubts